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Scientific Program - Timetable

Time	Sunday 22	Monday 23	Tuesday 24	Wednesday 25	Thursday 26	Friday 27
9:00			Contributed sessions (15 in parallel)	Plenary Lecture Moritz Diehl	Contributed sessions (15 in parallel)	Contributed sessions (14 in parallel)
10:00		Registration		von Mises prize lecture		
11:00			Coffee Break	Coffee Break	Coffee Break	Coffee Break
11:30			Plenary Lecture Thomas Böhlke	General Assembly	Plenary Lecture Ferdinando Auricchio	Contributed sessions (11 in parallel)
12:30			Lunch	Lunch	Lunch	
13:00		Opening				
13:30		Univ. Chorus Performance				Closing
14:00		Prandtl Lecture Keith Moffatt	Plenary Lecture Enrique Zuazua	Contributed sessions (15 in parallel)	Plenary Lecture Daniel Kressner	
15:00		Plenary Lecture Giovanni Galdi	Plenary Lecture Nikolaus Adams		Plenary Lecture Stanislaw Stupkiewicz	
16:00	Registration pre-opening	Coffee Break	Coffee Break Poster session	Coffee Break	Coffee Break Poster session	
16:30		Minisymposia & Young Reseachers' Minisymposia (10 in parallel)	Contributed sessions (14 in parallel)	Contributed sessions (15 in parallel)	Contributed sessions (15 in parallel)	
18:30			Public lecture Francesco D'Andria			
19:00		Opening reception at Castle of Charles V				
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21:00						

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S21: Mathematical image processing

Over the last decade mathematics has become the cornerstone in Signal and Image processing ranging from various methods for signal reconstruction to modelling of imaging modalities over its classical disciplines compression, denoising, segmentation, and registration to feature extraction. The used methodologies include such diverse fields as harmonic analysis, inverse problems, variational analysis, mathematical statistics, partial differential equations, optimization, approximation theory and sampling theory. The aim of this section is to foster interdisciplinary collaboration and the development of new directions in mathematical signal and image processing spawned from the interaction of various mathematical communities.

On learning reaction diffusion models

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In this work, we study the propblem of learning nonlinear reaction-diffusion models for image restoration. We extend conventional nonlinear reaction diffusion models by several parametrized linear filters as well as several parametrized influence functions. We propose to train the parameters of the filters and the influence functions through a loss based approach. Experiments show that our trained nonlinear reaction-diffusion models largely benefit from the training of the parameters and finally lead to the best reported performance on common test datasets for image restoration.

Adaptive Total Variation Regularization

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Due to its property to encourage piecewise constant solutions, Total Variation (TV) plays an important role as regularizer for various image restoration tasks. In order to also deal with piecewise affine structures, higher order variants such as Total Generalized Variation (TGV) [1] have been proposed.

To even further improve the restoration quality of TV regularization, various adaptive methods have been proposed in literature. Among these, we find the class of direction-dependent TV methods, which we refer to as *anisotropic* TV methods.

The main focus of the adaptive methods are e.g. the preservation of contrast, edges, corners, slopes and fine image structures. To steer the adaptivity, additional information is required. We therefore can distinguish between data-driven approaches, where this information is taken from the input image or additional data, and solution-driven approaches, where the adaptivity is depending on the solution of the problem.

We also remark that while the classical TV regularization is convex some of the proposed approaches in literature yield non-convex problems. While in view of the restoration quality these non-convex methods show advantages, their theoretical and numerical treatment comes with some difficulties.

In our talk we will first give an overview over TV variants lately proposed in literature and then discuss a novel method we have proposed recently [2, 3]. This strategy combines advantages of convex and non-convex approaches. It is based on a fixed-point problem, where the unknown on the one hand is the sought solution to the problem and on the other hand steers the adaptivity. Solving this problem numerically results in a sequence of convex problems, while the regularization asymptotically mimics a non-convex regularization. Reformulating the fixed-point problem as a Quasi-Variational Inequality Problem (QVIP) enables us to provide theoretical results for existence and uniqueness of the fixed point.

Finally, we exemplarily consider the applications of denoising, (non-blind) deblurring and inpainting.

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Joint Motion Estimation and Image Reconstruction

Hendrik Dirks, Martin Burger, Carola-Bibiane Schönlieb

Modern microscopes are able to visualize even the smallest biological processes within cells. One example is intracellular flow where the flow dynamics within a single cell are visualised by a sequence of microscopic images over time. Extracting accurate information on very short timescales, however, comes with a lack of spatial quality of the images in terms of resolution and noise.

One possibility to counteract this and enhance the image sequence is to use the fact that the recorded images are connected by motion of objects over time. In this talk we present a model that couples flow information into an image inpainting/denoising problem and simultaneously calculates the flow pattern between timesteps. Inspired by the ROF model [1] for image reconstruction

$$\min_u \frac{1}{2} \|Ku - f\|_2^2 + \alpha \|\nabla u\|_1 ,$$

with a general linear operator K that might represent for example a convolution, and a $TV - L^1$ model for motion estimation [2]

$$\min_{\boldsymbol{v}} \|u_t + \nabla u \cdot \boldsymbol{v}\|_1 + \beta \|\nabla \boldsymbol{v}\|_1 ,$$

we want to combine these models now, to similarly reconstruct an image sequence u and the underlying motion field $\boldsymbol{v}$. Therefore we use a constrained variational model

$$\begin{aligned} \min_{u,\boldsymbol{v}} \int_0^T & \frac{1}{2} \|Ku - f\|_2^2 + \alpha \|\nabla u\|_1 + \beta \|\nabla \boldsymbol{v}\|_1 \, dt \\ \text{s.t. } & u_t + \nabla u \cdot \boldsymbol{v} = 0. \end{aligned} \tag{1}$$

In the analytical part necessary conditions for the existence of minimizers for (1) are presented. In the end we introduce a primal-dual [3] minimization scheme and show some of our results.

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Regularization methods for flow fields with smooth transitions and sharp edges

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Cells in Motion – Cluster of Excellence, University of Münster[4]

With this contribution we present ideas for variational flow models that combine the abilities to recover smooth transitions as well as sharp edges inside of a flow field. Our aim is to estimate the intracellular and extracellular flows of migrating cells, which are expected to be sharp at the cell boundaries, but at least partially smooth inside of a cell. We study regularizers that are composed of total variation and of terms that allow for smooth transitions, i.e. we want to minimize functionals of the following type:

$$J(u) = \lambda \| f_t + (\nabla f)^T \cdot u \| + \min_v \left\{ \alpha_0 \, \|\nabla u - v\| + \alpha_1 \, R(v) \right\},$$

where f denotes the brightness of two consecutive images and u the vector field we are looking for. The first term of the functional J(u) is the optical flow constraint and the second term depicts the total variation part of the regularization. Possible choices for $R(v)$ that we want to focus on are $\frac{1}{2}\|v\|_2^2$ or $\|\nabla v\|_1$. The latter leads us to total generalized variation regularization, which is up to now mainly used for static models like denoising and reconstruction. To enhance the exactness of the recovery, we analyze the effect of Bregman iteration on the results and the influence of different norms for the data fidelity.

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Regularisation by Circular Hough Transform

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In 1962, Paul Hough patented a method to recognise straight lines in images [3]. The Hough transform, named after its inventor, was then further developed and generalised by Duda and Hart in 1972 [2]. They extended it to other parametrised curves and first applied it to circle detection. Nowadays, the circular Hough transform (CHT) is a well-established and widely-used shape detection method. In its continuous formulation, it can be written as a path integral along a circle for a function $f(x_1, x_2)$, $x_1, x_2 \in \mathbb{R}$:

$$\int\int f(x_1,x_2)\delta(r-\sqrt{(x_1-c_1)^2+(x_2-c_2)^2})\,dx_1dx_2,$$

where (c_1, c_2) are the centre coordinates and r is the radius.

This contribution makes use of the CHT in a different context. In particular, it is utilised in regularisation terms occurring in connection with variational methods for image processing tasks such as denoising or segmentation. The idea arose while performing automatic detection of circularly shaped mitotic cells in phase contrast microscopy images by recognising maxima of the CHT, which had been applied to an edge image, as well as cell tracking, where the CHT might also replace TV regularisation terms in the segmentation functionals. Since in most applications we are only looking for a small number of circular objects, the CHT image should accordingly contain a few peaks and hence the sparser it is, the clearer becomes the circular shape in the original image. This observation motivates integrating the CHT in regularisers for images containing circular objects.

In recent years, total variation regularisation has proven to be a convenient choice in many applications. In image denoising, the most popular example is the ROF model [4], which can be written in the following discrete form:

$$\min_u \frac{\lambda}{2}\|u-f\|_2^2+\|\nabla u\|_1,$$

where f is a given noisy image and u denotes the optimal denoised image to be obtained. Inspired by the fact that edge detection is performed before applying the CHT in the context of circle recognition and also due to the natural sparsity of edge images, we penalise the ℓ_1-norm of the CHT of the gradient image. The corresponding model reads

$$\min_{u,d} \frac{\lambda}{2}\|u-f\|_2^2+\|Hd\|_1+\chi_0(\nabla u-d),$$

where H denotes the CHT written as a discrete linear operator and χ_0 is the characteristic function of the set $\{0\}$. Note that the operator $H\nabla$ was split using the auxiliary variable d. In order to minimise this energy numerically, we use the first-order primal-dual algorithm proposed by Chambolle and Pock [1].

We present several results using synthetic as well as real-world examples. Although in most cases TV denoising can obtain better reconstructions with respect to PSNR or SSIM, it is remarkable that it also performs well using the proposed regularisation and it can be observed that the staircasing effect typical for TV denoising seems to be attenuated. Moreover, circular objects are smoothed and their histograms tend to be equalised, which can be advantageous for the task of cell mitosis detection and tracking as mentioned above. In addition, we discuss possible enhancements, e.g. obtained by combining TV and CHT regularisation or by using non-linear regularisation terms such as $\|H(|\nabla u|)\|_1$.

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Artifact-free variational MPEG decompression

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The MPEG (Motion Picture Experts Groups) video compression standard is one of the most well-known and widely-used methods to compress and store digital video data. The underlying concepts of MPEG compression, namely motion compensation and Block Discrete Cosine Transform (BDCT) encoding, are at the heart of almost any modern video compression method. MPEG achieves high compression rates which come, however, with the cost of loss of data due to quantization (rounding) of BDCT coefficients. This loss of data is responsible for disturbing artifacts in the decompressed movie.

In this talk, we present a variational method for improved MPEG decompression which is able to reduce compression-induced artifacts. By utilizing the information provided by the compressed MPEG file, our approach relies on the extraction of quantization intervals associated with the compressed data. These intervals must contain the "original data", i.e., the data prior to compression, and are used to describe the convex set D of all possible reconstructions. We then variationally decompress the video by minimizing a spatio-temporal regularization functional $\mathcal{R}$ subject to the data being contained in D.

As the decoding process, i.e., the mapping which takes the stored coefficients to the spatio-temporal video data, constitutes a linear operator A, the variational problem can be phrased as

$$\min_{d \in D} \mathcal{R}(Ad). \tag{1}$$

Our method conceptually works for all versions of MPEG compression, but is implemented for MPEG-2 compressed video data, as this version is still one of the standard encoding formats and can be seen as a realistic but still tractable blueprint for most video compression methods.

For regularization, we use the *Infimal Convolution of Total Generalized Variation* (ICTGV) functionals as introduced in [3]. The introduction of this functional is motivated by the observation that, for spatio-temporal regularization, the ratio between the spatial and temporal step-size − say λ − is not given a-priori and a combination of different ratios can be utilized to further improve reconstruction quality. Consider for instance the spatio-temporal second order *Total Generalized Variation* (TGV) [1] functional which is given as

$$\mathrm{TGV}^2_{\lambda,\alpha}(u) = \min_v \alpha_1 \|\nabla_\lambda u - v\|_1 + \alpha_0 \|\mathcal{E}_\lambda v\|_1, \tag{2}$$

where $\lambda > 0$ can be interpreted as a weight of the temporal derivatives appearing in the spatio-temporal gradient ∇_λ and symmetrized gradient $\mathcal{E}_\lambda$. This additional degree of freedom, which in fact defines a trade-off between spatial and temporal regularization, can be exploited to further improve reconstruction quality by optimally balancing between two choices of λ. This is realized by the infimal convolution of two second order TGV functional which is given as

$$\mathrm{ICTGV}^2_{\lambda,\beta,\alpha}(u) = \min_v \mathrm{TGV}^2_{\lambda_1,\alpha}(u - v) + \beta \mathrm{TGV}^2_{\lambda_2,\alpha}(v). \tag{3}$$

and used for regularization in the present work.

For the numerical solution of (1) we use the primal dual algorithm of [2] and present an implementation that covers all orders of spatio-temporal TGV regularization and infimal convolutions thereof. Our numerical experiments confirm that, using ICTGV regularization, we are able to significantly reduce compression artifacts in MPEG compressed videos and obtain a good reconstruction quality even at relatively high compression rates.

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Cartoon-Texture-Noise Decomposition with Transport Norms

Christoph Brauer, Dirk Lorenz
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We investigate the problem of decomposing an image into three parts, namely a cartoon part, a texture part and a noise part. We argue that norms originating in the theory of optimal transport should have the ability to distinguish certain noise types from textures. Hence, we present a brief introduction to optimal transport metrics and show their relation to previously proposed texture norms [1, 2, 3, 4]. We propose different variational models of the form

$$\min_{u,v} \quad \alpha F_u(u) + \beta F_v(v) + \gamma F_w(u^0 - u - v)$$

and investigate their performance. These models yield a decomposition of an observed image u^0 into cartoon u, texture v and noise $w = u^0 - u - v$. Our algorithmic approach exploits recent advances in non-smooth large-scale optimization [5, 6]. Moreover, inspired by [7] we illustrate the relations of the proposed models for cartoon-texture-noise decomposition to the theory of optimal transport.

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Asymptotics of Spatial Sparsity Priors

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Sparsity regularization in inverse problems based on minimizing the ℓ^1-norm is an important and versatile tool and can even be extended to realize more advanced a-priori information such as differently structured sparsity for unknowns being matrices. Regularization functionals, which promote sparse solutions, are indeed usually finite-dimensional. However, in many applications an infinite-dimensional modeling especially in the pre-image space seems to be more reasonable. Nevertheless, due to computational rather than application-related reasons, spatial sparsity is usually imposed on some spatial grid. Therefore, the establishment of an appropriate asymptotic theory behind these discrete approaches is of particular importance. An underlying asymptotic theory moreover permits an analysis independent of discretization and thus yields robustness.

This talk is devoted to the asymptotics of regularization methods applicable for instance to spatial sparsity regularization. We are interested in recovering continuum limits such as total variation regularization or recent approaches in the space of Radon measures, cf. for instance [1], [2] and [3]. Thereby, we will also obtain continuum limits, which were not understood up to now. In order to study the asymptotics of regularization functionals, we utilize Γ-convergence, which is a useful and reasonable framework for this purpose.

In so doing, we are able to compute some Γ-limits. We not only consider usual ℓ^p-norms for $p \geq 1$, but also analyze the asymptotics of the ℓ^0-"norm". On the basis of these insights, we moreover deduce some Γ-limits for certain types of mixed norms. In order to verify our results numerically, we furthermore consider the deconvolution of a sparse spike pattern for different discretizations as the step size of the grid becomes smaller.

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Image Warping via Optimal Transport with Sources

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In this contribution a new optimal transport approach for image warping is presented. The warp is obtained via the minimization of an action functional, which takes into account the transport costs and a source term. The source term locally measures the variation of the image density in L^1, which includes not only absolutely continuous sources with respect to the Lebesgue measure, but also singular measures. Existence of minimizing paths in the space of Radon measures is demonstrated. Furthermore, a robust and effective discretization is derived following the approach by Benamou and Brenier. Characteristic test cases underline the qualitative properties of this approach and selected applications show the potential for image matching, distance computation on the space of image, and weighted averaging of textures.

Non-rigid registration for pre–operative 3D surfaces and intra–operative 2.5D surfaces

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Image Guided Surgery (IGS) benefits from the fusion of different image data sources. In particular, the fusion of pre-operative and intra-operative information is helpful to the physicians. Due to the different image acquisition devices that are available before and during a surgery, registration of images of different dimensions is called for. For instance, [1] illustrates such benefits in the context of cranial neuronavigation by matching 2D intra–operative photographs with pre-operative MRI data sets.

In this talk, a novel non–rigid registration algorithm that matches pre–operative 3D surfaces and intra–operative 2.5D surfaces is presented, building on ideas proposed in [2] to register intra-oprative time–of–flight surfaces to pre-operative surfaces extracted from computed tomography for radiation therapy.

The goal of the proposed method is to find a non–rigid deformation that matches an intra–operative surface to a pre–operative one. Here, it is important that the deformation is defined on the intra-operative surface due to its lower dimension compared to the pre-operative surface. The registration problem is formulated as an energy minimization problem, in which the key roles are played by a matching energy term that involves the signed distance of the 3D surface to estimate the mismatch between the two surfaces and a regularization term for the deformation. As is usual for image registration approaches [3], the minimization of the energy is achieved through a multilevel coarse-to-fine strategy. On each level, a derivative based optimization approach is used for the numerical solution of the problem.

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Active-contours for image segmentation relying on non-stationary subdivision schemes

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Image segmentation is one of the primary steps in image analysis and visual pattern recognition. Active contours (also known as snakes) are well-known tools for image segmentation, belonging to the wider class of deformable models. An active-contour is essentially a curve or a surface that evolves within a 2D or a 3D image from some initial position, in order to detect the boundary of the object of interest. Active contours have become popular because they facilitate user interaction, not only during the initialization phase, but also during the minimization process of the energy function that governs the evolution of the curve/surface towards the boundary of the region to be segmented. The extensive research activity carried out in this field has resulted in many active-contour variants which differ both in the type of representation and in the choice of the energy function to be minimized. In this talk we present a new class of active-contours, never investigated so far, which describes the curve/surface in a discrete fashion by using a sufficiently refined polyline/polyhedral mesh obtained by the iterative application of non-stationary univariate/bivariate subdivision schemes. The subdivision schemes at the basis of our deformable models are non-stationary subdivision schemes at least C^1-continuous, with the property of translation invariance and the capability of perfectly outlining ellipses/ellipsoidal objects. A selection of illustrative examples in the context of biomedical images segmentation will demonstrate the excellent tradeoff between flexibility and efficiency that can be achieved by the newly derived active-contours.

Deterministic Sparse FFT Algorithms

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We propose deterministic stable FFT algorithms to compute a sparse vector $\mathbf{x} \in \mathbb{C}^N$ from its Fourier transformed vector $\widehat{\mathbf{x}} = \mathbf{F}_N \mathbf{x}$, where $\mathbf{F}_N$ denotes the Fourier matrix of order N. In case of nonnegative vectors being M-sparse, we need at most $\min\{M \log(N), N\}$ Fourier values in order to recover $\mathbf{x}$ and (for $M^2 < N$) at most $\mathcal{O}(M^2 \log N)$ arithmetical operations. Here, the sparsity M needs not to be known in advance but will be determined during the algorithm. If the considered vector $\mathbf{x}$ is not sparse, we obtain a usual FFT algorithm requiring $\mathcal{O}(N \log N)$ arithmetical operations.

In case of vectors $\mathbf{x} \in \mathbb{C}^N$ with small support M, we present a new sparse FFT using only $\mathcal{O}(M \log N)$ arithmetical operations if the support length M of $\mathbf{x}$ is a priori known. If the support length is not known in advance, the idea can be modified using Prony kind methods such that we need $\mathcal{O}(M \log M \log N)$ arithmetical operations.

The algorithms work iteratively, where each iteration step only involves the solution of a linear system of size at most M. We develop adaptive strategies to ensure that the coefficient matrix in the linear system is well-conditioned. For this purpose, we have to study Vandermonde matrices with knots on the unit circle.

Reconstruction of Multivariate Exponential Functions from Measurements

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Recovering an M-sparse sum of complex exponentials from equidistant measurements can be accomplished e.g. by using Prony's method or one of its numerical variants [1]. We want to reconstruct a d-dimensional signal

$$f(x) = \sum_{j=1}^{M} c_j \mathrm{e}^{\langle x, f_j \rangle}, \quad x, f_j \in \mathbb{C}^d, \quad c_j \in \mathbb{C}$$

by generalising those ideas to higher dimensions. In [2, 3] methods are introduced that show, how important subclasses of this multi-dimensional problem can be solved by reducing them to several one-dimensional problems. Only recently, another approach called superresolution was introduced in [4] that can also be generalised to more than one dimension.

Here we want to present a different approach based on Prony's method that involves the analysis of zero sets of d-variate polynomials. All the mentioned ideas share a significant amount of similarities that we would like to address.

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Ambiguities in one-dimensional discrete phase retrieval from Fourier magnitudes

Robert Beinert, Gerlind Plonka
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The phase retrieval problem has many applications in physics. For example, it occurs in crystallography, electron microscopy, astronomy, and optics. We consider the reconstruction of an unknown complex-valued signal $\mathbf{x}$ with finite support from the moduli of its Fourier transform $|\widehat{x}(\omega)|$. It is well known that the considered phase retrieval problem always has multiple solutions. For example, we can construct further solutions by rotating, shifting, or reflecting and conjugating a given solution of the problem. Besides these trivial ambiguities, there are also problems with further non-trivial ambiguities.

If we can describe the signal $\mathbf{x}$ as a convolution of elementary factors, we can determine all possible solutions of a phase retrieval problem – trivial or non-trivial. Furthermore, such a factorization always exists for signals of finite length.

There have been many different approaches to enforce uniqueness of the one-dimensional, real phase retrieval problem, especially the interference with known and unknown reference signals or the additional consideration of end points of the unknown signal. Most of these ideas can also be applied to the complex phase retrieval problem. In some applications, besides the moduli of the Fourier transform, the moduli of the unknown complex-valued signal are known.

With the aid of the representation of the ambiguities as convolution of elementary factors, we can show that this additional information about the unknown signal reduces the set of ambiguities such that every or almost every signal can be uniquely recovered up to some trivial ambiguities.

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Optimal mollifiers for the reconstruction of spherical images from circular means

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We are concerned with the reconstruction of spherical images from mean values along great circles or small circles. More precisely, we consider for circles $\mathcal{C}(\boldsymbol{\eta}, t) = \{\boldsymbol{\xi} \in \mathbb{S}^2 \mid \boldsymbol{\xi} \cdot \boldsymbol{\eta} = t\}$ on the two–sphere $\mathbb{S}^2$ the inversion of the operator

$$\mathcal{M} \colon C(\mathbb{S}^2) \to C(\mathbb{S}^2 \times [-1,1]), \quad \mathcal{M}f(\boldsymbol{\eta}, t) = \int_{\mathcal{C}(\boldsymbol{\eta},t)} f(\boldsymbol{\xi})\, \mathrm{d}\boldsymbol{\xi}$$

given discrete, noisy data $g_m = \mathcal{M}f(\boldsymbol{\eta}_m, t_m) + \varepsilon_m$, $m = 1, \ldots, M$, at points $(\boldsymbol{\eta}_m, t_m) \in \mathbb{S}^2 \times [-1,1]$ corrupted by white noise ε_m. Our approach combines the mollifier idea with a quadrature rule to derive an estimator of the form

$$f * \psi \approx f_\psi(\boldsymbol{\xi}) = \sum_{m=1}^{M} \omega_m g_m \Psi(\boldsymbol{\xi} \cdot \boldsymbol{\xi}_m, t_m),$$

where $\omega_m \in \mathbb{R}_+$, $m = 1, \ldots, M$ are some quadrature weights and the radial function $\psi \in L^2(\mathbb{S})$ with $\psi = \mathcal{M}^* \Psi$ is the mollifier. In our talk we will give optimal minimax rates

$$\inf_{\psi} \sup_{\substack{f \in H_s(\mathbb{S}^2) \\ \|f\|_s < S}} \mathbb{E} \, \|f - f_\psi\|_2^2$$

together with asymptotically optimal mollifiers ψ as $M \to \infty$ for the different sampling schemata $(\boldsymbol{\eta}_m, t_m) \in \mathbb{S}^2 \times [-1,1]$, $m = 1, \ldots, M$ provided that f belongs to a Sobolev ball. Finally, we illustrate our findings by numerical experiments and discuss fast algorithms for the forward as well as for the inverse transform that make use on the nonequispaced fast Fourier transform on the sphere.

A robust estimation method for camera calibration with known rotation.

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Extrinsic camera calibration is the process that yields the camera pose, i.e., its location and orientation with respect to a world reference frame. Usually its been calculated by estimating the epipolar geometry between two images and aggregating the results.

Epipolar geometry describes the relative geometry of two images depicting the same scene. It is encoded in a 3×3 singular matrix known as the fundamental matrix [3]. Estimating the fundamental matrix and thus the epipolar geometry, is a core ingredient for many of computer vision algorithm such as structure-from-motion [5], vision-based robot navigation [4] and even for intra-operative guidance [2].

A common practice is to use matching invariant features, e.g. SIFT, SURF etc. followed by a robust model-fitting algorithm. Typically, a RANSAC [1] like algorithm that samples a set of putative correspondences until an outliers-free set is found. The main weakness of RANSAC-like scheme is the requirement of sampling a valid set, where all are inliers, with a high probability. The number of sampling subsets that are needed is exponential with the subset's cardinality.

In this paper, we present an algorithm for relative pose estimation that exploited measurements data from inertial sensors, such as accelerometer, magnetometer and gyroscope which yield the camera orientation. We demonstrate that in case of known orientation the problem reduced to a 3-dimensional least-squares optimization problem, which can be readily implemented into a 3-points RANSAC scheme.

Experimental results on synthetic and real data assessed the accuracy and efficiency of our algorithm.

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Harmonic analysis of projectors

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The analysis of complicated data, say high-dimensional data, usually requires some sort of dimension reduction step. We shall develop a general framework for choosing suitable projectors in finite dimensional Euclidean space.

Indeed, the Grassmannian can be considered as the set of orthogonal projectors of fixed rank in the d-dimensional Euclidean space. Cubatures and designs on the Grassmannian have been well-studied in the recent literature. On the other hand, particular sets of projectors with potentially varying ranks have been used in signal processing under the name fusion frames. The relations between cubatures, designs, and fusion frames have already been investigated in the literature when the rank was held fixed. Here, we introduce cubatures and designs in unions of Grassmannians and discuss the relations towards fusion frames with varying ranks. We characterize cubatures and designs in unions of Grassmannians by means of the fusion frame potential matching a certain lower bound, and we present parametric families of symmetric designs in unions of Grassmannians.

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Perturbations of frame sequences and the effect on their duals

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We study small perturbations of (operator-valued) frame sequences and examine the canonical and alternate duals of original and perturbed sequence. It is proved that to each dual of the unperturbed frame sequence there is a dual of the perturbed one such that also the duals are "close to each other". Roughly speaking, the duals are stable under small perturbations.

We also study the perturbation effect on fusion frame duals – a concept which was recently introduced by Heineken et al. in [1]. It turns out that this kind of duality is much harder to tackle. However, we obtain a corresponding stability result in the finite-dimensional case.

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Construction of Multichannel Wavelets via Full Rank Subdivision Schemes

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Full rank vector subdivision schemes, in particular those which have an interpolatory nature, are connected to matrix refinable functions generating orthogonal multiresolution analyses for the space of vector-valued signals. Corresponding matrix wavelets (here called multichannel wavelets) lead to a quite natural "wavelet" tool for the analysis of vector-valued signals. Such kind of signals, which arise in several real world situations, like image processing (color images), finance, seismology, or biomedical applications (e.g. electroencephalographic data), cannot be processed using classical tools operating on vector-valued data component by component ignoring the possible relation between the components. In this talk, we discuss the properties of full rank interpolatory vector subdivision schemes, the connection with orthogonal matrix scaling functions, the concept of multichannel multiresolution analysis, and an algorithm for the explicit construction of orthonormal multichannel wavelets, together with some examples of applications.

Shearlet-Based Edge Detection: Flame Fronts and Tidal Flats

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Shearlets are wavelet-like systems which are better suited for handling geometric features in multi-dimensional data than traditional wavelet systems [1]. As such, it seems natural to use shearlets for edge detection. A novel method for edge detection which is in the spirit of phase congruency [2] but is based on a complex shearlet transform will be presented. Pixels are assigned values between 0 (not an edge) and 1 (sharp transition). Intermediate values indicate the presence of a smoother transition between different areas. Additionally, this approach to edge detection yields an approximate tangent direction of detected edges as a byproduct of the edge measure computation.

Two applications of the edge detection method will be discussed. The tracking and classification of flame fronts is a critical component of research in technical thermodynamics. Quite often, the flame fronts are transient or weak and the images are noisy. The standard methods used in the field for the detection of flame fronts do not handle such data well. Fortunately, using the shearlet-based edge measure yields dramatically better results.

The Wadden tidal flats are a biodiverse region along the North Sea coasts of the Netherlands, Germany, and Denmark. They are a UNESCO World Heritage site due to their ecological importance. Thus, surveying the delicate region and tracking the topographical changes are of utmost importance. One approach to mapping the area is to use pre-existing Synthetic Aperture Radar (SAR) images, supplemented with other information like water level [3]. Unfortunately, SAR data suffers from multiplicative noise as well as sensitivity to environmental factors, like how much wind was blowing across the surface of the water. The first large-scale mapping project of that type showed good results but only with a tremendous amount of manual interaction on top of the computerized image processing methods. In addition to weaknesses inherent in the methods used, there are many edges in the data which are not boundaries of the tidal flats but are edges of features like fields or islands. A method using a combination of the shearlet-based edge measure with a diffusion-based image segmentation will be presented.

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Classification of Edges Using Compactly Supported Shearlets

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We analyze the detection and classification of singularities of functions $f = \chi_B$, where $B \subset \mathbb{R}^d$ and $d = 2, 3$. It will be shown how the set ∂B can be extracted by a continuous shearlet transform associated with compactly supported shearlets. Furthermore, if ∂S is a $d-1$ dimensional piecewise smooth manifold with $d = 2$ or 3, we will classify smooth and non-smooth components of ∂S. This improves previous results given for shearlet systems with a certain band-limited generator, since the estimates we derive are uniform. Moreover, we will show that our bounds are optimal. Along the way, we also obtain novel results on the characterization of wavefront sets in 3 dimensions by compactly supported shearlets. Finally, geometric properties of ∂S such as curvature are described in terms of the continuous shearlet transform of f.

Approximation in variation for nonlinear Mellin integral operators

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Mellin analysis is well-known in approximation theory and Mellin operators are widely studied in the literature (see [13, 11] for an extensive theory, and, for other approximation results, [9, 8, 7, 4, 5]). The importance of Mellin operators is not only from a mathematical point of view, but also because of their applications in several fields. For example, they are connected with some problems of Signal Processing: indeed, Mellin analysis is very useful in situations in which, in order to reconstruct a signal, the samples are not uniformly spaced, as in the classical Shannon Sampling Theorem, but exponentially spaced (see, e.g., [10, 12]).

In this direction, we present the theory and some recent applications of Mellin integral operators: we will show how they can be used to approximate multivariate functions of bounded variation and we will also discuss some possible applications of such operators, together with other similar families of operators, to problems of image reconstruction.

In particular, we will study a family of nonlinear Mellin integral operators defined as

$$(T_w f)(\mathtt{s}) = \int_{\mathbb{R}^N_+} K_w(\mathtt{t}, f(\mathtt{st}))\frac{d\mathtt{t}}{\langle \mathtt{t} \rangle},\ \mathtt{s} \in \mathbb{R}^N_+,\ w > 0,$$

where $\{K_w\}_{w>0}$ is a family of kernel functions, $\langle \mathtt{t} \rangle := \prod_{i=1}^N t_i$, $\mathtt{t} = (t_1, \ldots, t_N) \in \mathbb{R}^N_+$, and $f : \mathbb{R}^N_+ \to \mathbb{R}$ is a function of bounded variation.

As pointed out before, our interest about approximation results in $BV-$spaces is due also to the important applications of such results in some problems of image reconstruction: indeed, the setting of $BV-$spaces is suitable in order to describe jumps of grey-levels of the image that correspond, from a mathematical point of view, to discontinuities. In this direction it is essential to consider the case of functions of several variables. For this reason we will use a new multidimensional concept of variation and, in order to treat Mellin integral operators, the most natural approach is to frame our study on $\mathbb{R}^N_+$ equipped with the log-Haar measure $\mu(A) := \int_A \langle \mathtt{t} \rangle^{-1}\, d\mathtt{t}$, where A is a Borel subset of $\mathbb{R}^N_+$. For references about approximation results in $BV-$spaces in multidimensional frame, see e.g. [6, 2, 1, 3]. The new multidimensional concept of variation that we will use was introduced in [5] and it was adapted to the setting of $\mathbb{R}^N_+$ from the multidimensional variation by Tonelli.

By means of this concept of variation, we first obtain several estimates for the family of integral operators $\{T_w f\}_{w>0}$, proving, for example, that they are well-defined and that they map $BV(\mathbb{R}^N_+)$ into itself. Then we prove the main convergence theorem, which states that

$$\lim_{w\to+\infty} V[T_w f - f] = 0,$$

whenever $f \in AC(\mathbb{R}^N_+)$ (the space of absolutely continuous functions on $\mathbb{R}^N_+$). Introducing suitable Lipschitz classes, we also study the problem of the rate of approximation. Moreover, in the particular case of Fejér-type kernels, we obtain that all the assumptions of our results are implied by the classical condition that the absolute moments of order α of the kernels are finite.

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Multivariate sampling Kantorovich operators: from the theory to the Digital Image Processing algorithm

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Applications to Approximation and to Signal Theory by means of family of sampling operators have been largely studied since $1980s$, in order to investigate possible extensions or generalization of the classical Wittaker-Kotelnokov-Shannon (WKS) sampling theorem, see e.g. [5, 1, 11]. The WKS sampling theorem provides an exact reconstruction formula for band limited, finite energy functions. In order to weaken the above assumptions on the signal being reconstructed, the German mathematician P.L. Butzer introduced the generalized sampling operators, see e.g., [4, 3]. These operators revealed to be very suitable to approximate (in some sense) continuous signals f by means of sampling series involving sample values $f(k/w)$, $k \in \mathbb{Z}$, $w > 0$. However, in real word cases, signals (such as images) are often discontinuous. For instance, gray scale images are characterized by jumps of gray levels in the areas close to the edges of images, and they can be modeled by multivariate discontinuous signals. Here, we present the theory and some applications to Digital Image Processing (D.I.P.) of the multivariate sampling Kantorovich operators (see e.g., [2, 7, 6]), defined by:

$$\text{(I)} \qquad (S_w f)(\underline{x}) := \sum_{\underline{k} \in \mathbb{Z}^n} \chi(w\underline{x} - t_{\underline{k}}) \left[\frac{w^n}{A_{\underline{k}}} \int_{R^w_{\underline{k}}} f(\underline{u})\, d\underline{u} \right], \qquad \underline{x} \in \mathbb{R}^n,$$

where $f : \mathbb{R}^n \to \mathbb{R}$ is a locally integrable function such that the above series is convergent for every $\underline{x} \in \mathbb{R}^n$. The function $\chi : \mathbb{R}^n \to \mathbb{R}$ is called a kernel, and it satisfies suitable properties. Moreover, $t_{\underline{k}} = (t_{k_1}, ..., t_{k_n})$ denotes vectors where each $(t_{k_i})_{k_i \in \mathbb{Z}}$, $i = 1, ..., n$ is a certain strictly increasing sequence of real numbers with $\Delta_{k_i} = t_{k_{i+1}} - t_{k_i} > 0$. Note that the sequences $(t_{k_i})_{k_i \in \mathbb{Z}}$ are not necessary equally spaced (irregular sampling scheme). The symbols $R^w_{\underline{k}}$ denote the sets of the form:

$$\text{(II)} \qquad R^w_{\underline{k}} := \left[\frac{t_{k_1}}{w}, \frac{t_{k_1+1}}{w} \right] \times \left[\frac{t_{k_2}}{w}, \frac{t_{k_2+1}}{w} \right] \times ... \times \left[\frac{t_{k_n}}{w}, \frac{t_{k_n+1}}{w} \right],$$

$w > 0$ and $A_{\underline{k}} = \Delta_{k_1} \cdot \Delta_{k_2} \cdot ... \cdot \Delta_{k_n}$, $\underline{k} \in \mathbb{Z}^n$. The sampling Kantorovich operators S_w (see also [12, 8, 13]) represent an L^1-version of the generalized sampling operators and they are very suitable to reconstruct not necessarily continuous signals, thus applications to image reconstruction can be deduced.

The main approximation results for the above operators are respectively a pointwise and uniform convergence theorem for bounded continuous and bounded uniformly continuous functions on $\mathbb{R}^n$. In this case, a result concerning the order of approximation can be obtained considering signals in suitable Lipschitz classes of the Zygmund-type (see [9, 10]). In order to obtain a convergence result for not necessarily continuous signals, the theory of multivariate sampling Kantorovich operators has been studied in the general setting of Orlicz spaces $L^\varphi(\mathbb{R}^n)$, where φ is a convex φ-function, see e.g. [4]. These spaces are very general and they include as noteworthy special cases the well-known L^p-spaces. Other important and useful examples of Orlicz spaces, for which the above theory can be applied, are provided by the interpolation and the exponential spaces.

In $L^\varphi(\mathbb{R}^n)$, the modular convergence of the family $(S_w f)_{w>0}$ to f is proved, being $f \in L^\varphi(\mathbb{R}^n)$. In the latter setting, in order to study the rate of approximation for the sampling Kantorovich operators, a suitable definition of Lipschitz classes can be introduced by using the modular functional of the space $L^\varphi(\mathbb{R}^n)$.

Based on the above approximation results, from the definition of the operators in (I) and for a suitable choice of the kernel function χ, an algorithm for image reconstruction and enhancement can be implemented.

A great importance in the above D.I.P. algorithm is the choice of the multivariate kernel function χ, which in general influence the rate of approximation and the performance of the sampling Kantorovich operators. A procedure that allow us to construct multivariate kernels by the product of univariate kernels satisfying the assumptions of the above theory can be showed. Examples of one-dimensional kernels are the well-known Fejér's kernel, the central B-spline of order $k \in \mathbb{N}$, the Jackson-type kernels and many others, see e.g., [2, 7].

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Stockwell Transform and Applications to Image Analysis

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Let f be a signal of finite energy, then

$$(\mathrm{S}\, f)\,(b,\xi) = (2\pi)^{-1/2} \int_{\mathbb{R}} e^{-2\pi \mathrm{i}\, t\xi} f(t)\, |\xi|\, e^{-(t-b)^2\xi^2/2} dt, \quad b,\xi \in \mathbb{R}$$

is called the Stockwell transform (S-transform) of the signal f. It was first introduced in 1996 by R. G. Stockwell *et al.* in connection to seismic problems, see [2]. Roughly speaking, we can say that $|\,(\mathrm{S}\, f)\,(b,\xi)|$ represents the energy of the signal f at a given time b and at a given frequency ξ. Clearly, this statement is not really precise due to the Heisenberg Uncertainty Principle.

The S-transform lies in between the Short Time Fourier Transform and the Wavelet Transform. It shares with the Short Time Fourier Transform a clear time-frequency representation. In fact, it can be seen as the Fourier transform of the signal f cut by an analyzing window, in this case a Gaussian. The main difference is that the window is frequency dependent. This change implies that, as the frequency increases, the width of the analyzing window shrinks. This idea is consistent with the Nyquist Theorem. On the other hand, this feature shows a clear link with the Wavelet Transform, but in this case we have not a time-scale domain, rather a real time-frequency plot. The computational cost of the S-transform of a signal of length N is $\mathcal{O}\left(N^2 \log N\right)$, the same of the Short Time Fourier Transform and of the of Continuous Wavelet Transform.

In collaboration with L. Riba [1], we have studied a modified S-transform, called DOST, introduced again by Stockwell in [3]. This transform gained interest after the fast algorithm introduced by Y. Wang and J. Orchard [4], which reduced drastically the complexity to $\mathcal{O}\left(N \log N\right)$. This improvement enlarged the applications of the DOST to problems in image processing as, for example, denoising and compression. The DOST decomposes a signal f of length N into N-coefficients $f_{p,\tau}$. Each coefficient $f_{p,\tau}$ represents the time-frequency content of the signal in a certain time frequency box. The properties of the transform imply that, for low frequencies, we have very high precision in frequency, and low precision in time; while, for high frequencies, we have high precision in time and low precision in frequency. Translated into image analysis, this means that the DOST is able to detect with high precision edges.

We will finally present some results obtained in collaboration with L. Riba, L. Sambuelli and S. Pirro. We applied the DOST to noise reduction in the framework of GPR (Ground Penetrating Radar) data. In particular to GPR analysis of Archaeological sites. This type of data is usually affected by a large number of noises, therefore the post processing plays a central role. The traditional tools in this field are based on Fourier Analysis, while time-frequency methods are not usually considered.

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Applications of Approximation Theory to thermographic images in earthquake engineering

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In this talk we present some applications of an approximation process by means of a family of sampling type operators to thermographic images, (see e.g., [2, 1, 3, 4]).

The mathematical theory of these operators shows how it is possible to reconstruct and to enhance multivariate signals, such as images. In particular, we are able to reconstruct images taken from thermographic survey of masonry walls, and to enhance their quality. Thermographic images are largely used to make diagnosis and monitoring buildings. Moreover these images are used both to assess effective dimensions of structural elements and for masonry texture, i.e. the mutual arrangement of the blocks (made of stones and/or bricks) and mortar joints inside the wall portion analyzed.

For the image texture we use digital images algorithms that perform as follow: first of all we apply a median filter to the image using a suitable mask, then the image is converted into a black and white image by means of a suitable thresholding, morphological operators are used to enhance the quality of the separation of the phase such as the closure of the area to eliminate salt-and-pepper noise and erosion and dilation operators to smooth the contours of the inclusions.

We will show that the image texture obtained by the application of our algorithms is more realistic from an engineering point of view and more fitting to the real structure and therefore allows us to make an accurate analysis of the building.

The reconstruction methods are used to estimate the mechanical characteristics of the masonry walls of a real-world case-study. These mechanical properties allows us to analyze the response of a masonry structure under seismic actions in terms of modal analysis.

Our interest is in the applicability of the proposed procedure and the advantages that can be achieved in comparison with more traditional approaches. In particular our model suggests a method to overcome some difficulties that arise when dealing with the vulnerability analysis of existing structures and allows us to estimate the mechanical characteristics of the masonries using non-destructive methods, (see e.g., [5, 6]).

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A generalization of the Zak transform with applications in sampling theory and physics

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The Zak transform on $\mathbb{R}^d$, $d \in \mathbb{N}$, has many applications in signal processing and quantum mechanics – it provides a time-frequency decomposition of a signal [1] and serves as a tool to study quantum mechanical systems with lattice symmetries [2]. Generalizations of the Zak transform to locally compact abelian groups and beyond have been studied in [3, 4].

In this contribution, a framework will be presented that includes these generalizations, while also being applicable to the action of unimodular automorphism groups on a quite general class of locally compact groups, for example. This generalized Zak transform is based on a Weil formula for quasi-invariant measures on double coset spaces. It naturally has applications for the sampling of signals at orbits of groups that act on the underlying space and provides complete sets of functions to study physical systems with respect to their symmetries. In particular, an application in radiation design will be presented.

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Multivariate Generalized Sampling Type Series: estimates of pointwise convergence

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This contribution presents a Kantorovich multivariate version of the generalized sampling series of a function $f : I\!\!R^N \to I\!\!R$, namely

$$(K_{p(n)}f)(x) = \sum_{k\in \boldsymbol{Z}}^{N} \varphi(p(n)x-k)\left[\langle p(n)\rangle \int_{Q_{k,n}} f(u)du\right], \quad x \in I\!\!R^N,$$

where $p : I\!\!N \to I\!\!R^N$ is a vector valued function satisfying suitable assumptions, φ is a suitable kernel and

$$Q_{k,n} = \prod_{j=1}^{N}\left[\frac{k_j}{p_j(n)}, \frac{k_j+1}{p_j(n)}\right],$$

with $p = (p_1, \ldots, p_N)$, $k = (k_1, \ldots, k_N)$ and for any vector $a = (a_1, \ldots, a_n)$, we put $\langle a\rangle = \prod_{j=1}^{N} a_j$. We study some asymptotic formulae for the pointwise convergence of the operators $K_{p(n)}f$ to f. Then we introduce certain suitable linear combinations of these operators in order to obtain a better order of the pointwise convergence. Finally we apply the theory to the multivariate Bochner-Riesz sampling series. These results were obtained in [2]. Related operators were also studied in [4] and [5], in the frame of Orlicz type spaces in which extensions of the one-dimensional theory introduced in [1] are outlined.

More recent generalized versions are also introduced in [3], in which a Durrmeyer type approach is used.

These studies have direct applications to signal processing, in particular to image reconstruction.

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A New Blind Source Separation Numerical Technique

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Digital Document Restoration (DDR) consists of a set of processes finalized to the visual and aesthetic improvement of a virtual reconstruction of the document. By DDR a document can be analyzed without deteriorating it. Main degradation types are: *Bleed-through*, that is a physical phenomenon due to the ink infiltration from the opposite side of a page; *show-through*, that is an effect due to the scanning process and the paper transparency. We deal here with the problem of estimating the original sources from two data mixtures of these sources produced by the bleed through or the show through effect. This problem belongs to the *Blind Source Separation* (BSS) class of problems.

We assume that the mixture model is linear. This assumption is not always verified in the real cases, anyway it is essential to construct the basement of more complex models. The data images that we consider are the front and back of a degraded document. The inverse problem consists in estimating both the source images and the mixture matrix, given the observed data. This problem is a well known ill-posed problem [2], that is, in some cases, the solution neither exists, nor is unique, nor can be stable in presence of noise.

With the aim of restored the well-position of the problem, different techniques are proposed in the literature; in particular many of them that imposed the orthogonality of the results [1, 3]. We present here a new technique that try to find the best factorization of the covariance matrix of the data. The experimental results confirm the goodness of such a technique.

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Geometric Means of Toeplitz Matrices by Positive Parametrizations

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We consider the problem of finding means of geometric type of Hermitian positive definite matrices which are also Toeplitz, i.e., constant along their diagonals.

One might be tempted to use the geometric mean of positive definite matrices, the Karcher mean [3], which is obtained as the barycenter in a suitable Riemannian geometry on the positive definite matrices. Unfortunately, the geometric mean is not well suited for averaging Toeplitz matrices, since the mean of Toeplitz matrices may not be Toeplitz.

To overcome this difficulty, two different approaches have been introduced:

- an axiomatic approach, which produces a structured geometric mean, satisfying most of the properties which are desirable for a geometric mean, but giving up some of them, in order to enforce the structure [4];
- a constructive approach, which defines a new metric on Toeplitz matrices and choose as the geometric mean the barycenter there, said to be the Kähler mean [1].

The latter mean can be obtained using a parametrization of the $n \times n$ positive definite Toeplitz matrices through n complex numbers with positive real part, moreover it can be uncoupled considering just n scalar barycenters [4].

By using different parametrizations of the Toeplitz matrices, we generalize this approach obtaining different definitions of Toeplitz means with the same features as the Kähler mean but with better computational properties. We investigate further the asymptotic behaviour of these means with respect to the functional interpretation of sequences of Toeplitz matrices of increasing size [5].

The problem under investigation arises in radar signal processing, where the use of differential geometry in mathematical statistics (information geometry) has given recently important updates [2]. In particular, in the design of radar systems in the presence of a randomly moving background (clutter), such as the see, a way to proceed is to do repeated measures of a suitable autocorrelation matrix relative to the noised signal. These matrices are Toeplitz and positive definite and need to be averaged, and their average should verify some properties of the geometric mean.

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A unifying theory for convergence of linear sampling operators in Orlicz spaces

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Sampling operators play a crucial role in Signal and Image Processing. The *generalized sampling series* [1]

$$S_w f(x) = \sum_{k\in\mathbb{Z}} \chi(wx-k) f\left(\frac{k}{w}\right) \tag{1}$$

where χ is a continuous kernel with compact support over $\mathbb{R}$ is just an example of a discrete operator which can be used to reconstruct a function (signal) from its sampling values $f\left(\frac{k}{w}\right)$. However, the series (1) fails to be appropriate when f is no longer continuous. Obvious practical reasons led to the introduction of a sampling series of *Kantorovich* type [2], namely

$$K_w f(x) = \sum_{k\in\mathbb{Z}} \chi(wx-k) w \int_{\frac{k}{w}}^{\frac{k+1}{w}} f(u)du. \tag{2}$$

In (2), the values $f\left(\frac{k}{w}\right)$ are replaced by the averages

$$w \int_{\frac{k}{w}}^{\frac{k+1}{w}} f(u)du,$$

with the result of reducing the so-called *time-jitter errors* and obtaining the convergence for a wider class of functions (e.g., for functions which lie in an Orlicz space $L^{\varphi}(\mathbb{R})$). However, other sampling operators have been defined to treat signals when other types of errors appear, for instance the *round-off errors*.

In order to manage simultaneously the theories of discrete linear sampling operators, we introduce the following general form of the sampling series

$$V_w f(x) = \sum_{k\in\mathbb{Z}} \chi(wx-t_k) L_{t_k/w} f, \tag{3}$$

where $(L_{t_k/w}) : M(\mathbb{R}) \to \mathbb{R}$ is a family of continuous functionals having as domain the set $M(\mathbb{R})$ of measurable real functions [6]. The values t_k are introduced to consider *nonuniform sampling*.

We discuss the construction and analyze the convergence of the series (3), including the general setting of Orlicz spaces $L^{\varphi}(\mathbb{R})$ (i.e., for not necessarily continuous functions).

Motivated by the developments in [7], the construction can be further generalized to the setting of topological groups, by introducing the operators

$$T_w f(z) = \int_{t\in H} \chi(z-h_w(t)) L_{h_w(t)} f d\mu(t), \tag{4}$$

where H, G are locally compact topological groups, $(h_w) : H \to G$ is a family of homeomorphisms, $(L_{h_w(t)}) : M(G) \to \mathbb{R}$ is a family of continuous operators having as domain the set $M(G)$ of measurable functions defined on the topological group G and $\mu(t)$ is the Haar measure on H. This approach allows to include integral operators, to generalize convolution and Mellin operators and to discuss multidimensional signals as well (see, e.g., [3, 4]).

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Digital image processing algorithms for diagnosis in arterial diseases

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Computed Tomography images (C.T.) are currently part of the routine procedure in medical diagnostic techniques. The use of CT images can be focused on the evaluation of occlusion rate of arterial vessels in presence of atheromas, atherosclerosis, and thrombi's formation process. The correct individuation of the morphology of these arterial anomalies allows specialists to diagnose the risk rate for the health of patients and to decide for surgical stent implants [3]. Due to no radio opacity of the blood, the CT image acquisition procedure often does not allow the doctors to identify by visual analysis the anatomy of the occlusive structures in the arterial vessels: in these cases the use of radio-opaque contrast medium becomes necessary. On the other hand the introduction of radio-opaque contrast medium is critical in patients affected by kidney dysfunctions and is anyway not healthy in any case.

Multivariate sampling Kantorovich operators [2, 1, 9, 5, 4, 10] and subsequent wavelet analysis allow to emphasize the pathological arterial vessel morphology improving the visual diagnosis even without contrast medium introduction. The previous family of operators are suitable for studying not necessarily continuos signals/images as the ones involved in the medical field [7]. Increasing the sampling rate and choosing an appropriate kernel we are able to enhance the images under consideration [6].

The standard in medical digital images (Digital Imaging Communication in Medicine, D.I.CO.M.) preserves linear and area measurements even using differently compression algorithms.

CT images are gray-scale bidimensional matrices and it's possible to use the discrete two dimensional wavelet transform (2D D.W.T.) [8], deriving it from the continuos wavelet transform (C.W.T.) defined as:

$$\text{(II)} \qquad \gamma(s,\tau) = \int f(t)\psi^*_{\tau,s}(t)dt \quad , \quad t \in \mathbb{R}$$

where s is the scale factor, τ is the translation and ψ^* is the complex conjugate of the mother wavelet function, defined as:

$$\text{(III)} \qquad \psi(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right) \quad , \quad \tau, s \in \mathbb{R}$$

and assuming in the DWT case the form:

$$\text{(IV)} \qquad \psi_{jk}(t) = \frac{1}{\sqrt{s_0^j}}\psi\left(\frac{t-k\tau_0 s_0^j}{s_0^j}\right) \quad , \quad j,k \in \mathbb{N}$$

The 2D DWT together with the enhancement performed on CT images allow to extract more precisely the contours of the pathological occlusive structures while DICOM standard preserves the real geometrical dimensions, all resulting in a more precise medical diagnosis.

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Nonnegative Tensor Grid Decomposition

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The topic of Nonnegative Matrix Factorization (NMF) consists in finding two nonnegative matrices $U \in \mathbb{R}^{m\times k}$ and $V \in \mathbb{R}^{k\times n}$ such that the product UV is the best approximation of a given nonnegative matrix $M \in \mathbb{R}^{m\times n}$ for some criterium. In most applications, it is desirable to obtain a factorization where k is much smaller than n, since this allows the approximation of the columns of M as linear combinations of the fewer columns of U. In terms of segmentation, the nonnegativity of the elements in these matrices provides an interesting interpretation of the factors U and V: the columns of U can be interpreted as the templates in the segmentation, while the entries in V hold the probability that a certain column of M belongs to a such a template.

A generalization of this problem replaces the matrices M and U by a tensor grid in which each element M_{ij} and $U_{i\ell}$ is now a positive definite matrix of size μ, with $i = 1,\dots,m$, $j = 1,\dots,n$, and $\ell = 1,\dots,k$. The factorization itself has recently been generalized by Xie et al. [2] by minimizing the cost function

$$\mathcal{E}(U,V) = \sum_{i=1}^{m}\sum_{j=1}^{n} \|M_{ij} - \sum_{\ell=1}^{k} U_{i\ell} v_{\ell j}\|_F^2,$$

where $\|.\|_F$ is the Euclidean (Frobenius) norm.

The above model for factorizing the tensor grid M implicitly assumes Euclidean geometry for the combination of U and V and the distance between M and this combination. However, the set of positive definite matrices has a natural intrinsic geometry which has proven advantageous in the modelling of application data [3]. Applying this geometry to the method for combining U and V and to the distance function between M and the combination results in our new cost function

$$\mathcal{R}(U,V) = \sum_{i=1}^{m}\sum_{j=1}^{n} \delta^2\left(M_{ij}, K(v_{1j},\dots,v_{kj}; U_{i1},\dots,U_{ik})\right),$$

where δ is the intrinsic distance of the positive definite matrices and K represents the weighted Karcher mean [1], for which $K(v_{1j},\dots,v_{kj}; U_{i1},\dots,U_{ik})$ is defined as the unique minimizer over the set of positive definite matrices of the function $f(X) = \sum_{\ell=1}^{k} v_{\ell j}\delta^2(X, U_{i\ell})$.

Finally, as a compromise between the geometry of the positive definite matrices and the speed of the Euclidean model, we discuss the Log-Euclidean setting. The main idea of this setting is to retract positive definite matrices to the set of symmetric matrices using the matrix logarithm, and afterwards proceeding as in the Euclidean case. Doing so results in a third cost function

$$\mathcal{L}(U,V) = \sum_{i=1}^{m}\sum_{j=1}^{n} \|\log(M_{ij}) - \sum_{\ell=1}^{k} v_{\ell j}\log(U_{i\ell})\|_F^2.$$

All three models are applied to test data where both low rank (k) image reconstruction and data segmentation are investigated, in terms of both accuracy and computational time.

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Poisson Noise Removal from High-Resolution Electron Micrographs based on periodic Block-matching

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In this talk, we propose a denoising technique optimized for images from Scanning Transmission Electron Microscopy (STEM). This is an imaging technique that provides sub-ångstrom, atomic resolution images of crystalline structures by moving a focused electron probe over a very thin material along a regular grid and measuring the electrons exiting on the other side of the material within a certain annulus. In many applications, the ability to extract valuable information such as atom positions, from the acquired images, is severely obstructed by low signal-to-noise ratios due to limitations to the electron dose during acquisition. These limitations result from noticeable damage inflicted on the material by the electron beam.

The presented denoising strategy is based on the Block-matching and 3D filtering algorithm (BM3D) [1] with improvements tailored to the special features of atomic-resolution electron micrographs of crystals limited by Poisson noise. The major improvement consists of an optimized block-matching strategy that predicts the periodic structure of the observed crystal and distributes adaptively refined non-local search windows throughout the image [2]. Furthermore, it is shown that such a true non-local block-matching strategy can lead to a highly non-uniform distribution of the number of aggregated estimates available in each pixel, resulting in poor estimation in image regions where the distribution of block estimates is sparse. Possible remedies for this deficiency are discussed.

Numerical experiments were conducted with simulated low dose STEM images of crystals with different grid structures. The results show an increase in peak signal-to-noise ratio (PSNR) of $2-4$ dB when using our method instead of the original BM3D algorithm. Our method also improves electron microscopy related quantitative measures, such as detection fraction and precision (standard deviation of detected inter-atomic distances). The precision in the estimates of simulated low-dose electron micrographs of perfect crystals produced by our proposed algorithm are within $7-15$ pm. These results are competitive with the best reported precisions in the literature for single shot STEM images [3]. Furthermore, a new measure, called fidelity, is introduced, which evaluates the discrepancy between the atom centers detected in the estimate and the ones detected in the ground truth. Our experiments indicate that this error can be significantly reduced by denoising electron micrographs prior to atom detection.

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Application of learning algorithms for colour recognition on underwater images

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Objects look very different in the underwater environment compared to their appearance in sunlight. The main reason is that the penetration of light through seawater is highly dependent on the wavelength of the light [4]. Suspended particles in the water can further decrease the overall quality of underwater images [1, 2].
High quality images with correct colouring simplify the detection of underwater objects and may allow the use of visual SLAM algorithms developed for land-based robots underwater. Hence, image processing is required to obtain images of high quality and correct colouring. Over the last decade significant progress has been made in this direction.
Current algorithms focus on the colour reconstruction of scenery at diving depth [2, 5]. This has the advantage that a significant part of sunlight is still present and different colours can still be distinguished although they may be tainted due to light filtered through seawater. Other approaches require a known geometric relations between the camera and the object [6], assume a simple relation between colours under different lighting conditions using Beer's law [4], or require manual work [3].
Unfortunately, at greater depth the filtering is much stronger such that Beer's law is not sufficient. Also different colours are strongly tainted and can no longer be distinguished. To solve this the main factor of interest for the influence on colour is the penetration depth d_p which is a strong function of the wavelength λ.
The approach suggested here uses a special light source with a defined wavelength to illuminate the environment. The defined wavelength is characterised by a high penetration depth in water. Particularly, wavelengths of $450 - 550\,nm$ are of interest in ocean water [8, 7]. It is also possible to use a light source emitting white light as distant objects will also have a colour change due to absorbtion in seawater that will result in a similar appearance of the surroundings as with the defined wavelength source.
Within this study the image taken under these special lighting conditions is fed into a learning algorithm which was calibrated using images of objects with known colour and structure. The colours in the image are then transformed by the algorithm to obtain an image as seen under white light in air.
The new procedure currently uses the original image without any additional filtering. In the next step prefilters will be included into the algorithm to remove reflection effects and to smooth the brightness in the image. Additionally, the pre-processing after Bazeille et al. [1] will be tested. It may also be of interest to include the information of neighbouring pixels for the calculation of the colour to remove single pixels which are not transformed correctly.
The developed algorithm and procedure can be applied also for other applications, where other ranges of wavelengths are used, to obtain the image as seen under white light from data under different lighting conditions.

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Edge detection based on fractional order differentiation and its application to railway track images

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Recurrent standard measurement tasks are often necessary and scheduled within the development cycle of railway vehicles. Many of these measurement tasks need to be done in a very early stage of the project or in the offer period of a light rail vehicle. Thereby, a main focus is the recognition of the track geometry and its disturbances without significant influence on the daily scheduled operation. By the means of this measured data it is possible to calculate the structural stress within significant assembly parts of the train [1]. This track geometry detection process is realized by a novel contactless modular measurement system. Within the development process of this system, many challenges according to the camera based railway recognition occurred. In this context, a eCRONE (extended Contour Robuste d'Ordre Non Entier) operator - an extended version of the non-local CRONE edge detection operator introduced by [2] - based on Grünwald-Letnikov definition of fractional order differentiation is presented. In recent years, fractional calculus is getting more and more important to various applications such as [3], [4] and [5]. Equation (1) shows the one-dimensional CRONE operator ${}^{\mathrm{CR}}\underset{\leftrightarrow}{\mathrm{D}}{}_x^{\nu} f(x)$, which is based on a superposition of a causal and a non-causal derivative of a function $f(x)$ of fractional order $\nu \in \mathbb{R}$.

$$ {}^{\mathrm{CR}}\underset{\leftrightarrow}{\mathrm{D}}{}_x^{\nu} f(x) = (\delta_N x)^{-\nu} \sum_{j=0}^{N-1} \underbrace{\frac{\Gamma(j-\nu)}{\Gamma(-\nu)\Gamma(j+1)}}_{g_j} \left[f\left(x - j\delta_N x\right) - f\left(x + j\delta_N x\right)\right]. \tag{1} $$

Within (1), the gamma function is denoted by Γ and the Grünwald coefficients are given by g_j. A reformulation of (1) yields the following filter kernel matrix as

$$ {}^{\mathrm{CR}}\underset{\leftrightarrow}{\mathbf{D}}{}_x^{\nu} = \begin{pmatrix} \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & & 0 & 0 & 0 & & 0 & 0 \\ g_m & g_{m-1} & \cdots & g_1 & 0 & -g_1 & \cdots & -g_{m-1} & -g_m \\ 0 & 0 & & 0 & 0 & 0 & & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots \end{pmatrix}. \tag{2} $$

Based on (1) and (2), a two-dimensional filter kernel, which takes the local neighborhood of the actual processed pixel into account, is presented in the paper. This significantly improves the detection performance and the stability to noise related to classical operators like Sobel, Prewitt etc. Furthermore, a description how the eCRONE detector response leads by the means of the discrete Radon transform to a mathematical description of parametric geometries like lines or circles within the responses is given.

Additionally, some first results of a practical application of the above mentioned algorithms within a real measurement campaign are presented respectively.

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